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A RESEARCH ON THE INTERVAL ESTIMATION OF PAIRWISE WEIGHT RATIO IN DATA ENVELOPMENT ANALYSIS

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Abstract

Traditionally, the Data Envelopment Analysis (DEA) efficiency evaluation model uses the mathematical linear programming to find a set of weights that can make their own efficiency value greatly for the Decision Making Units (DMU).Therefore, in the traditional model of DEA, there are generally unreasonable phenomena, including "Variable Weight is 0" and multiple "Efficient DMUs." In order to break through the irrational phenomena, this article uses the concept of AR Model (Thompson, 1986) and combines the Expert Weights Method (EWM) (Bao, 2013) to develop a new Model, which is the Interval Estimation of Pairwise Weight Ratio, EWM_AR Model. At the same time, this article proves a theorem which shows how the EWM_AR Model avoids the problem of generating the Formula of Weight Limit when dealing with the Interval Estimation of Pairwise Weight Ratio.

According to the research, after incorporating the Relevance of Each Variable, EWM_AR Model in this article produces two major achievements: (1) In terms of the weight limits, the shortcomings of the traditional "Variable Weight is 0" can be improved; (2) It can eliminate the shortcomings of the traditional multiple "Efficient DMU."

Keywords: Data Envelopment Analysis; Region-Assurance Method; Expert Weighted Method

Introduction

Traditionally, the DEA's efficiency evaluation model uses the mathematical linear programming to find a set of weights that can make their own efficiency value greatly for DMU. However, the traditional model of DEA generally has the irrational phenomena of "Variable Weight is 0" and the multiple "Efficient DMU."

In order to break through the irrational phenomena, this article combines the concept of "AR Model and the theoretical of Expert Weights Method" to set a new model of the upper and lower limits of the weight. This new model is referred to the Interval Estimation of Pairwise Weight Ratio, EWM_AR Model. At the same time, this article proves a theorem which shows how the EWM_AR Model avoids the problem of generating the Formula of Weight Limit when dealing with the Interval Estimation of Pairwise Weight Ratio.

This article will be divided into five sections: the first section with introduction, the second section with gives an example of the importance of the Reasonable Weight Limit, the third section with introduces the research methods of EWM_AR Model, the fourth section with is the empirical results and discussion, the conclusion.

The Importance of the Reasonable Weight Limit

For the Weight Limit, the DEA-CCR Model is freely developed as laissez-faire. That is, when a variable of DMU_k is optimal relative to other DMU*s*, it's possible that only this variable of DMU_k is given a weight value, and other variables are zero (Doyle and Green, 1994). The severity of the phenomena will lead to "Variable Weights is 0" and more than one "Efficient DMU." Although there have been related improvement researches proposed in many scholarly literatures, the irrational phenomena will still generate. For the irrationality of weight limits, the following three examples of traditional models are used to illustrate the importance of Reasonable Weight Limits.

- Assurance Region Model (Thompson *et al*., 1986)

In the AR model, the "Weight Ratio" of "Paired Variables" can be limited to a reasonable range, so that the situation of "Extreme Distribution" can be avoided:

$$\alpha_{ij}^{L} \le v_i/v_j \le \alpha_{ij}^{U}\ ;$$
$$\beta_{ij}^{L} \le u_i/u_j \le \beta_{ij}^{U} \qquad (2\text{-}1)$$

where, α_{ij}^{L} and α_{ij}^{U} are the upper and lower limits of the weight ratio of the input variables; β_{ij}^{L} and β_{ij}^{U} are the upper and lower limits of the weight ratio of the output variable.

However, the drawback of AR model lies in the weight limit of the equation (2-1), because it does not explain how the upper and lower limits of these weight ratios are determined.

- Cone Ratio Model (Charnes *et al*., 1989)

In general, the CR mode can be divided into two categories: (1) Weight Limit for Intersection Form: $c_1u_1 + c_2u_2 + \ldots + c_su_s \ge 0$. (2) Weight Limit for Sum Form: $u =$

$A^T\omega$; where, A^T is the input-output matrix; ω is the marginal substitution rate.

The drawback of CR model is to limit the Weight to a closed convex cone, but how to define the Reasonable Boundary Range of the Convex Cone is not explained.

- The Theorem Proposed by Saaty for the Upper Limit of Variable Weights (2008)

The mathematical formula of the theorem, proposed by Saaty, is as follows. If there are n DMUs, m input variables, and s output variables, the upper boundary of weight (u_r, v_i) is
$u_r^* \geq 1/max\{y_r\}$;
$v_i^* \geq 1/max\{x_i\}$ (2-2)
where, $r = 1,2,\ldots,s$; $i = 1,2, \ldots ,m$.

Here, the data set of Hadad (2008) (such as Appendix A, Table A-1) will be used as an illustrative case. However, after applying (2-2), the result turned out to be:

$u_1^* \geq 0.000012117 \fallingdotseq 0$;

$u_2^* \geq 0.000002817 \fallingdotseq 0$;

$u_3^* \geq 0.000020665 \fallingdotseq 0$.

By the weight values, approaching 0, of these three output variables, it can be known that Saaty's Weight Ceiling Model still has its insurmountable drawbacks.

Looking at the descriptions of the above three traditional models, we can realize the importance and the key impact of the Reasonable Weight Limits. However, the newly developed EWM_AR Model is based on the AR Model and combines the theoretical methods of the EWM to set a new model of the upper and lower limits of the weight. At the same time, it is confirmed in the research results of this article that if the weighting conditions of the Relevance of Each Variable are included in the efficiency evaluation, it will obviously improve the irrational phenomena that the traditional model generally possess.

The Research Method of EWM_AR Model

Although many scholars and literature have proposed related improvement researches, the irrational phenomena of the DEA traditional model are still widespread. The reason is that most improvement researches focus mainly on the collection of data. Basically, the collection of these materials can be considered as samples needed for the efficiency evaluation, and it can only be effectively applied to that research.

However, if there is a slight deviation in the above mentioned data collection, it will result in a certain degree of error results for the Weight Limit. Therefore, this article is no longer based on the data collected, but emphasizes the importance of each variable itself. That is to say, by combining the "AR Model concept and the theoretical method of EWM," a new research method is produced. This new method refers to EWM_AR Model. The following are the research methods of EWM_AR Model:

Step1. After adding the theoretical method of the Relevance of Each Variable of EWM Model, the average number $\bar{x}_i$ and standard deviation S_i required for the

99.7% Confidence Interval Weight Value v_i of the Variable Weight are calculated.

Step2. According to the Law of Large Numbers, the limit formula of the weight ratio of the 99.7% Confidence Interval Weight Value v_i of the Variable Weight is calculated:

$$v_i^L \le v_i \le v_i^U \quad (3\text{-}1)$$

$$\frac{v_i^L}{v_k^U} \le \frac{v_i}{v_k} \le \frac{v_i^U}{v_k^L} \quad (3\text{-}2)$$

where, $v_i^L = \bar{x}_i - 3S_i$ be the lower limit of v_i ; $v_i^L = \bar{x}_i + 3S_i$ be the upper limit of v_i ; $k = 1, 2, \dots, m$ (m is the number of variables).

After the two equations (3-1) & (3-2) is rearranged, the main mathematical formula of EWM_AR Model can be obtained:

$$v_i^L \le v_i \le v_i^U$$

$$v_k^L \le v_k \le v_k^U$$

$$\left(\frac{v_i^L}{v_k^U}\right) v_k \le v_i \le \left(\frac{v_i^U}{v_k^L}\right) v_k \quad (3\text{-}3)$$

Assuming that there are m variable weights, when setting the weight ratio range, it usually generates $m*(m-1)$ formula of Weight Limits. However, this article proves a theorem which shows how EWM_AR Model avoids the problem of generating the Formula of Weight Limit when dealing with the Interval Estimation of Pairwise Weight Ratio.

$$2C_2^m = 2 \times \frac{m!}{2!(m-2)!}$$

$$= \frac{m!}{(m-2)!}$$

$$= m(m-1) \quad (3\text{-}4)$$

among them, the larger the m value, the more the formula of Weight Limit.

Theorem: Assuming that there are m variable weights, you only need to calculate the (m-1) number generated by the formula of the weight limit of (3-3), and the rest will be included.

Proof: Suppose the Variable Weight is v_k, and let the upper and lower limits be $k^L \le v_i \le k^U$.

$$\text{And let } \frac{v_i^L}{v_j^U} = \left(\frac{v_i}{v_j}\right)^L, \ \frac{v_i^U}{v_j^L} = \left(\frac{v_i}{v_j}\right)^U \quad (3\text{-}5)$$

where, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, m$.

Substituting (3-5) into (3-3), you can get:

$$\left(\frac{v_i}{v_j}\right)^L = \frac{v_i^L}{v_j^U} \le \frac{v_i}{v_j} \le \frac{v_i^U}{v_j^L} = \left(\frac{v_i}{v_j}\right)^U \quad (3\text{-}6)$$

The right side of the inequality (3-6) $= \frac{v_i^U}{v_j^L}$

$$= \frac{v_i^U}{v_k^L} \times \frac{v_k^L}{v_j^L} = \left(\frac{v_i}{v_k}\right)^U \times \left(\frac{v_k}{v_j}\right)^L$$

$$= \left(\frac{v_i}{v_k}\right)^U \times \left(\frac{v_j}{v_k}\right)^U$$

The left side of the inequality (3-6) $= \left(\frac{v_i}{v_j}\right)^L$

$$= \frac{v_i^L}{v_k^L} \times \frac{v_k^L}{v_j^L}$$

$$= \left(\frac{v_i}{v_k}\right)^L \times \left(\frac{v_k}{v_j}\right)^L \quad (3\text{-}7)$$

Finally, after substituting (3-7) into (3-6), you can get:

$$(\frac{v_i}{v_k})^L \times (\frac{v_k}{v_j})^L \leq \frac{v_i}{v_j} \leq (\frac{v_i}{v_k})^U \times (\frac{v_j}{v_k})^U \quad (3\text{-}8)$$

It can be known from (3-8) that the weight v_j of any variable j and the upper and lower limits of the ratio of the weight v_i of the variable i are covered by the ratio range of v_i and v_k. That is, it is no longer necessary to add these weight ratio ranges. Therefore, the theorem is proved.

The Empirical Results and Discussion

In order to prove that EWM_AR Model can eliminate the irrationality of the traditional model, “Smart Mobile Phone Brand Loved by Consumers” is selected as a research topic. 11 experts related to this topic are invited to provide their insights about efficiency assessments according to their professional cognition. However, the verification methods for EWM_AR Model are divided into two stages: the Implementation Method and the Results Discussion.

The first stage of the Implementation Method included the Condition Setting of the Object of the Expert Interview, and the Main Work Content of the Interview Process. (See Appendix B)

After expert interviews for three times, the results of the first stage of verification included: (1) the revision of the research topic, “The Evaluation Forecast of How to Purchase the Next Smart Phone”; (2) the evaluation variables of five non-off Specifications were summarized, including the preference of brand culture, appearance and texture design, technological innovation ability, maintenance (service) quality, and average selling price; (3) 13 smart phones were summarized as DMUs; (4) the experts provided professional cognitive evaluation data for 13 DMUs and 5 evaluation variables, as shown in Table 4-1.

The second stage of the Results Discussion included the Weight Estimation and Efficiency Assessment for the CCR Model, and CW Model and the newly developed EWM_AR Model. As shown in Table 4-2a~Table 4-2c and Table 4-3.

Looking at the data comparisons in Tables 4-2a through 4-2c, it is sufficient to prove that EWM_AR Model cannot only break through but also improve the irrational phenomenon, "Variable Weight is 0". Furthermore, as it can be seen from Table 4-3, the efficiency evaluation of the traditional model generally has two relative deficiencies: (1) Each DMU has the characteristic of $\theta_{EWM\_AR} < \theta_{CW} \leq \theta_{CCR}$, which proves that EWM_AR Model is more Objectivity; (2) Both θ_{CCR} and θ_{CW} have multiple "Efficient DMUs" problems of inaccurate discrimination. In contrast, EWM_AR Model, after incorporating the theoretical approach of the Relevance of Each Variable, proves that it can eliminate the shortcomings of traditional "Efficient DMUs."

Conclusion

The newly developed EWM_AR Model in this article combines the "AR Model concept and the theoretical method of Expert Weight Method (EWM)" to set a new model of the upper and lower limits of

Table 4-1. The appraisal data provided by the experts for the five appraisal variables

DMU	Brand Culture Preference Level y_1	Appearance and Quality Feeling Design y_2	Technological Innovation Capability y_3	Quality of Maintenance and Service y_4	CP value (Specification/ Price) y_5
*1.*ASUS	30.2	26.0	27.4	28.5	30.0
2. BENTEN	13.1	12.7	14.3	14.2	18.4
*3.*Huawei	24.0	23.5	26.0	24.2	25.8
*4.*HTC	28.8	29.9	28.6	27.3	26.5
5. Lenovo	19.2	20.0	21.1	21.8	23.0
*6.*LG	22.3	23.7	24.2	21.4	21.5
*7.*OPPO	26.7	27.5	26.6	27.1	28.5
*8.*SAMSUNG	26.5	29.3	29.5	24.4	25.0
9. SONY	30.4	29.4	28.0	26.4	27.8
*10.*Nokia	30.2	29.7	27.0	28.1	30.9
*11.*Xiaomi	24.5	26.7	26.5	25.4	28.6
*12.*ZTE	14.5	14.8	14.8	16.8	19.5
13.Apple	35.6	36.8	35.8	30.3	20.7

Table 4-2a. Weight values for variables obtained in CCR Model

DMU	v	u_1	u_2	u_3	u_4	u_5
1	1	0.0079	0	0	0.0182	0.0081
2	1	0	0	0	0	0.0324
3	1	0	0	0.0186	0	0.0161
4	1	0	0	0.0186	0	0.0161
5	1	0	0	0.0085	0.0156	0.0107
6	1	0	0	0.0186	0	0.0161
7	1	0	0	0.0085	0.0156	0.0107
8	1	0	0	0.0186	0	0.0161
9	1	0.0215	0	0	0	0.0114
10	1	0	0	0	0	0.0324
11	1	0	0	0.0186	0	0.0161
12	1	0	0	0	0	0.0324
13	1	0	0.0272	0	0	0

Table 4-2b, weight values for variables obtained in CW Model

v	u_1	u_2	u_3	u_4	u_5
1	0	0	0.0186	0	0.0161

Table 4-2c, Weight values of variables obtained in EWM_AR Model

v	u_1	u_2	u_3	u_4	u_5
1	0.00653	0.00554	0.00560	0.00633	0.00829

Table 4-3, DMU efficiency values obtained by CCR, CW and EWM_AR respectively

DMU	The research method of CCR & CW		The research method of EWM_AR
	θ_{CCR}	θ_{CW}	θ_{EWM_AR}
*1.*ASUS	1	0.9930	0.9236
2. BENTEN	0.5955	0.5623	0.4783
*3.*Huawei	0.8994	0.8994	0.7994
*4.*HTC	0.9591	0.9591	0.9062
5. Lenovo	0.7667	0.7630	0.6829
*6.*LG	0.7967	0.7967	0.7260
*7.*OPPO	0.9553	0.9540	0.8833
*8.*SAMSUNG	0.9517	0.9517	0.8621
9. SONY	0.9690	0.9688	0.9156
*10.*Nokia	1	1	0.9468
*11.*Xiaomi	0.9537	0.9537	0.8540
*12.*ZTE	0.6311	0.5893	0.5274
13.Apple	1	1	1

the weight. Moreover, in the research results of this article, it is found that if the weight limit condition of the Relevance of Each Variable is added to the efficiency evaluation, EWM_AR Model cannot only break through but also improve the " Variable Weight is 0" and a number of "Efficient DMU" irrational phenomena. This means that the “Interval Estimation of Pairwise Weight Ratio, EWM_AR Model will provide a more objective and unbiased efficiency evaluation model. The research results show that there are two main results of EWM_AR Model: (1) Looking at the data comparisons in Tables 4-2a through 4-2c, it is sufficient to prove that EWM_AR Model cannot only break through but also improve the irrational phenomenon, "Variable Weight is 0"; (2) From the phenomenon of multiple “Efficient DMUs” in Table 4-3, it can be proved that EWM_AR Model can eliminate the shortcomings of traditional "efficient DMUs."

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Appendix A

Table A-1, Table1 in the literature of Hadad (2008)

DMU	Input x_1	Input x_2	Output y_1	Output y_2	Output y_3
1	675	72	53017	220017	16535
2	148	12	24127	35748	905
3	193	8	16116	62865	465
4	366	16	32411	124478	1483
5	448	40	33577	145554	9433
6	108	4	11056	37252	1398
7	415	38	33492	131009	5753
8	618	70	49631	226015	23849
9	215	23	24239	85459	7265
10	499	17	54379	225010	2765
11	394	14	32234	129435	1393
12	535	75	51688	189197	9403
13	403	23	32030	153140	6435
14	207	32	24631	96605	5069
15	639	55	56016	213304	7416
16	949	71	71766	304311	28735
17	321	15	32358	110128	1193
18	858	40	68113	287528	48391
19	315	10	22136	91151	3763
20	378	26	31661	155705	2726
21	984	100	72187	323055	18932
22	984	76	82527	354925	21532
23	911	46	75698	306468	24056
24	618	30	58338	230186	9647

Appendix B

The Condition Setting of the Object of the Expert Interview

In line with the needs of the research topics in this article, and through friendly inquiry in a random manner, the conditions of the interviewees are finally set as follows: (1) interviewees should have more than two different brands of mobile phones within 6 years, and the price of each mobile phone should at least exceed NT$10,000; (2) interviewees should at least use not only mobile phones 6 hours but also 4 different types of software platforms a day.

The Main Work Content of the Interview Process

* The main job of the first interview was to collect the applicable Variables for the implementation efficiency assessment.

During the interview, everyone was aiming at 14 High-End Smart Phones with a price ranging from NT$10,000 to 20,000. In the end, a total of eight variables belonging to the specifications of smart phones were collected, including CPU, Memory, Camera, Screen, Battery, Warranty, and Optimization.

* The main job of the second interview was to amend the number and name of the Variables.

Some interviewees believe that if the naming of the variable is based on the Specifications of Smart Phones, it cannot be objectively evaluated. Therefore, 11 interviewees suggested adjusting the naming direction of the variable name, and summarized the number of evaluation variables into five, including Brand Culture Preference Level, Appearance and Quality Feeling Design, Technological Innovation Capability, Quality of Maintenance and Service, CP Value (Specification / Price).

* The main task of the third interview was to adjust the name of the research topic and to amend the Variables again.

As a result of the revision of the second interview, it is not entirely based on the consumer's position. Therefore, the interviewees agreed to change the issue name and the change of the evaluation again from the perspective of the consumer.